ISOMORPHISM BETWEEN CERTAIN SYSTEMS OF SIMPLE LINEAR GROUPS.

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1. In an article in the BULLETIN for July, 1899, giving the known finite simple groups, I made the conjecture that the simple quaternary hyperorthogonal group $HO(4, p^{2n})$ in the $GF[p^{2n}]$ was isomorphic with the second hypoabelian group $SH(6, 2^n)$, the orthogonal group $O(6, p^n)$, or the group $NS(6, p^n)$, according as p^n is of the form 2^n, $4l-1$, or $4l+1$ respectively. For the case* $p^n = 2$, and for the case† $p^n = 3$, I have proven the conjecture true by setting up abstract groups holoedrically isomorphic with the linear groups in question. The calculations were necessarily long, so that the method of procedure would scarcely be adapted to the case of general p^n. From the correspondence of generators established in these two special cases, I have been led to the short proof for the general case given in this paper. The proof is based upon the theory of the second compound of a linear homogeneous group, as developed in the BULLETIN for December 1898, and in the *Transactions* for January, 1900, pp. 91–96. In place of the hyperorthogonal group $HO(4, p^{2n})$, I employ the holoedrically isomorphic hyperabelian group‡ $HA(4, p^{2n})$, which contains as a subgroup the simple abelian group $A(4, p^n)$. The calculations appear to me to possess considerable elegance.

2. THEOREM.—*According as* -1 *is a square or a not-square in the* $GF[p^n]$ $(p > 2)$, $HA(4, p^{2n})$ *is holoedrically isomorphic with the group* $NS(6, p^n)$ *or* $O(6, p^n)$.

Corresponding to every substitution of the subgroup $A(4, p^n)$ there is a senary linear homogeneous substitution of the second compound H with the two invariants

$$(1) \quad \phi \equiv Y_{12}Y_{34} - Y_{13}Y_{24} + Y_{14}Y_{23}, \qquad Z \equiv Y_{12} + Y_{34}.$$

To the hyperabelian substitution, in which I belongs to the $GF(p^{2n})$,

$$I \equiv \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I^{-p^n} & 0 & 0 \\ 0 & 0 & I^{-1} & 0 \\ 0 & 0 & 0 & I^{p^n} \end{bmatrix}$$

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* *Proc. of the Lond. Math. Soc.*, vol. 31, pp. 30–68.

† *Transactions*, vol. 1, No. 3 (July, 1900).

‡ *Proc. Lond. Math. Soc.*, vol. 31, pp. 30–68.