# ON A DEFINITIVE PROPERTY OF THE COVARIANT.

BY MR. C. J. KEYSER.

(Read before the American Mathematical Society at the Meeting of April 29, 1899.)

THE general homogeneous entire polynomial of degree $n$ in $k$ variables $x$ may be denoted by

$$F_n(x_1, x_2, \cdots, x_k) \equiv \sum c_{e_1 e_2 \cdots e_k} x_1^{e_1} x_2^{e_2} \cdots x_k^{e_k}$$

where $e_1 + e_2 + \cdots + e_k = n$. Let

$$\varphi_n(\xi_1, \xi_2, \cdots, \xi_k) \equiv \sum \gamma_{e_1 e_2 \cdots e_k} \xi_1^{e_1} \xi_2^{e_2} \cdots \xi_k^{e_k}$$

represent the polynomial into which $F$ is converted by the substitutions

$$\begin{aligned} x_1 &= \lambda_{11}\xi_1 + \lambda_{12}\xi_2 + \cdots + \lambda_{1k}\xi_k, \\ x_2 &= \lambda_{21}\xi_1 + \lambda_{22}\xi_2 + \cdots + \lambda_{2k}\xi_k, \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ x_k &= \lambda_{k1}\xi_1 + \lambda_{k2}\xi_2 + \cdots + \lambda_{kk}\xi_k, \end{aligned}$$

where the $\lambda$'s are subject to a single restriction : their determinant $D$ shall not assume the value zero.

If there is such an entire homogeneous polynomial

$$\psi_m(x_1, x_2, \cdots, x_k) \equiv \sum h_{e_1 e_2 \cdots e_k} x_1^{e_1} x_2^{e_2} \cdots x_k^{e_k},$$

where $e_1 + e_2 + \cdots + e_k = m$ and where each coefficient $h$ is an entire homogeneous polynomial of degree $p$ in the coefficients $c$ of $F$, that

$$\psi_m(\xi_1, \xi_2, \cdots, \xi_k) \equiv M \cdot \psi_m(x_1, x_2, \cdots x_k),$$

the $\gamma$'s entering the left member of the identity as the $c$'s enter $\psi_m$ of the right member, then $\psi_m(x_1, x_2, \cdots, x_k)$ is named covariant or invariant of $F$ according as $m > 0$ or $= 0$.

Supposing such a function $\psi$ to exist, it remains to determine the nature of the factor $M$. The $\xi$'s and the $\gamma$'s being linear respectively in the $x$'s and the $c$'s, the two members of the identity in question are, apart from the factor $M$, each of degree $m$ in the $x$'s and of degree $p$ in the $c$'s. It follows that $M$ is a function of the $\lambda$'s only. $M$ is, more-