ON SINGULAR POINTS OF LINEAR DIFFERENTIAL EQUATIONS WITH REAL COEFFICIENTS.*

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LET us consider the equation

$$(1) \qquad \frac{d^n y}{dx^n} + p_1 \frac{d^{n-1} y}{dx^{n-1}} + p_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + p_n y = 0,$$

in which the coefficients $p_1, p_2, \cdots, p_n$ are throughout a certain interval $a < x < b$ continuous real (but not necessarily analytic) functions of the real variable x. By a solution of (1) we shall understand any function of x which together with its first $n-1$ derivatives is single valued and continuous throughout the interval $a < x < b$ and at every point of this interval satisfies (1). It is well known that there is one and only one solution of (1) which at an arbitrarily chosen point of the interval in question has together with its first $n-1$ derivatives arbitrarily chosen values. The object of the present paper is to consider the behavior of these solutions as we approach one end of the interval. It will clearly be sufficient if we confine our attention to the point a.

The simplest case would be that in which all the coefficients $p_1, p_2, \cdots, p_n$ approach finite limits as x approaches a. Then would come the case in which, although this is not true, none of these coefficients become infinite as x approaches a. Without considering separately these possibilities we will go on at once to a more general case which includes them as special cases.

We will say of a function $f(x)$ that it is integrable up to the point a if $\int_c^a f(x)dx$ (where $a < c < b$) converges; *i. e.*, if

*The only investigations with which I am acquainted concerning the singular points of linear differential equations whose coefficients are not assumed to be analytic, are contained in two papers by Kneser, *Crelle*, vols. 116, 117. These papers deal with a certain class of *irregular* points, to use the terminology suggested in the present paper. The singular point in question is taken at infinity.