ON A REGULAR CONFIGURATION OF TEN LINE PAIRS CONJUGATE AS TO A QUADRIC.

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AT the meeting of the London Mathematical Society, June, 1898, I showed a model of ten lines in space, each intersecting three others perpendicularly.* Subsequently Professor Study, seeing the model, remarked that there should be an analogous configuration of ten pairs of lines conjugate as to a quadric. This new configuration can be established as follows:

(1) Consider a ruled quadric H, and denote by Q an inscribed quadrilateral formed by two right generators and two left generators. The diagonals of Q are a pair of conjugate lines, say P. Thus Q determines P.

(2) Call two pair of generators harmonic when they cut another generator in harmonic pairs of points. Then two harmonic pairs of right generators will cut any conic of the surface H at the ends of conjugate chords, and the left generators through these ends will also be harmonic pairs. Hence calling two inscribed quadrilaterals Q and $\overline{Q}$ harmonic when their right generators and also their left generators are harmonic pairs, we can say that *harmonic quadrilaterals determine meeting conjugate pairs*, where we mean that each of the one pair meets each of the other.

(3) Now two quadrilaterals Q and Q' have a common harmonic quadrilateral; therefore two conjugate pairs are met by a third conjugate pair; that is, the two lines which met the four are themselves conjugate.†

(4) Take now three conjugate pairs P_1, P_2, P_3. Let the pair meeting both P_2 and P_3 be P_1'; thus we have three new pairs P_1', P_2', P_3'. Let the pair meeting P_1 and P_1' be P_1''; thus we have three new pairs P_1'', P_2'', P_3''. It is to be proved that these are met by one conjugate pair.

Replace the pairs by inscribed quadrilaterals, and the matter being merely one of harmonic constructions consider

* See *Proc. Lond. Math. Soc.* vol. 29.

† We set aside the special case in which the two pairs are met by an infinity of lines, which arises when the two quadrilaterals have the same right (or left) generators.