rational fraction is decomposed into partial fractions—these and other points are worthy of mention.

Taking the two volumes together, they form a valuable work of reference to the college teacher of calculus. To an unusual degree they give just what the student should know. The French sparkle is perhaps missing, but we are well satisfied with the German accuracy, thoughtfulness and thoroughness.

EDWARD B. VAN VLECK.

WESLEYAN UNIVERSITY, *May*, 1895.

TRIANGULAR NUMBERS. *

In a review † of Blater's Table of Quarter Squares, Mr. J. W. L. Glaisher, in 1889, referred to the convenience that would be afforded by a table of triangular numbers, and said that the only extensive published table of such numbers known to him was that by E. de Joncourt, published at the Hague in 1762, giving the values of $\frac{1}{2}n(n+1)$ from $n = 1$ to $n = 20{,}000$. Letting S_n represent the n^{th} triangular number, the sum of the n numbers up to and including n, or $\frac{1}{2}n(n+1)$, the application of a table of triangular numbers to facilitate multiplication is seen from the formula, $ab = S_{a-1} + S_b - S_{a-b-1}$. This formula shows the chief advantage claimed for the use of triangular numbers in preference to quarter squares—that the table need only extend as far as the highest number it is to be used to multiply. Thus, such a table need be only half as large as a table of quarter squares that may be used to multiply the same numbers by taking the difference between the quarter squares of the sum and difference of the factors. On the other hand the method of quarter squares requires but two instead of three tabular entries, and may be modified ‡ so as not to use an argument exceeding the larger factor; but in that case three tabular entries are required. A modification of the use of triangular numbers is also applicable to reduce the number of entries to two, but then we may need to use an argument greater than either factor.

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* Projet de Table de Triangulaires de 1 à 100,000, etc.; A. ARNAUDEAU (Paris: Gauthier-Villars et Fils, 1896).

† Reprinted in the *Journal of the Institute of Actuaries*, London, Jan. 1890.

‡ $ab = 2\left[\frac{a^2}{4} + \frac{b^2}{4} - \frac{(a-b)^2}{4}\right]$.