This theorem now admits of manifold applications. It gives us first the transcendency of e, if we regard $C_1, C_2, \ldots$ $z_1, z_2, \ldots$ as integral rational numbers. It gives further the transcendency of π. For from the equation $1 + e^{i\pi} = 0$ it follows by I. that $i\pi$, and consequently π, can not be algebraic.

It follows further that

For every algebraic number x, except $x = 0$, $X = e^x$ is a transcendental number.

For every algebraic X, except $X = 1$, every natural logarithm $x = \log X$ is a transcendental number.

For every arc which stands in an algebraically expressible relation x to the radius, except $x = 0$, $X = \sin x$ is a transcendental number.

This follows from I, since $2iX = e^{ix} - e^{-ix}$.

The same is true for the other trigonometric lines $\cos x$, $\tan x$, and for the chord $\frac{1}{2} \sin \frac{x}{2}$. To add one more result: The transcendental equation $\tan x = \alpha x$ for α algebraic has, except 0, only transcendental numbers for roots.

SHORTER NOTICES.

A Geometrical Treatment of Curves which are Isogonal Conjugate to a Straight Line with respect to a Triangle. Part I. By I. J. Schwatt, Ph. D., University of Pennsylvania. Boston, Leach, Shewell and Sanborn, [1895]. 8vo. 6 and 46 pp.

Four points serve to pair off all the points of a plane, if we take for any point its conjugate with respect to all conics through the four points. This is the reversible transformation which Durège (Ebene Curven dritter Ordnung) calls Steiner's Transformation. The absolute or circular points at infinity may be one of the point pairs; the conics are then rectangular hyperbolas, and the four basis points are orthocentric. Any two points OO' forming such a pair, when considered with reference to the diagonal triangle ABC of the four points, are said to be isogonal conjugates, for the reason that AO and AO' make opposite angles with AB and AC.

Transformation by isogonal conjugates is thus a special view of a simple transformation of great importance. In Dr. Schwatt's pamphlet, of which a continuation is in hand, the method is applied in particular to the discussion of the transformation of a straight line, which is, of course