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CR MAPPINGS OF FINITE MULTIPLICITY AND EXTENSION OF PROPER HOLOMORPHIC MAPPINGS

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1. Introduction. We shall describe some general theorems about CR mappings between three-dimensional manifolds which, among other results, imply that any proper holomorphic mapping $f: D \to D'$ between pseudoconvex domains in $\mathbf{C}^2$ with real analytic boundaries extends to be holomorphic in a neighborhood of the closure of D (Theorem 8). In case the domain D is strictly pseudoconvex, this result follows from the classical Lewy-Pinčuk reflection principle [**9**, **11**]. In case D' is strictly pseudoconvex, or in case D and D' are given by polynomial defining functions, f extends by [**2**]. In case the proper mapping f is biholomorphic, the extendability has been proved by Baouendi, Jacobowitz, and Treves [**1**]. The general case of a proper holomorphic mapping between weakly pseudoconvex domains which is not biholomorphic is more complicated because branching might occur. We have developed a method in the spirit of [**1**] which allows us to prove extendability at boundary points even if branching occurs (Theorems 3 and 6).

The mapping $f(z,w) = (z^2, w)$ which maps the domain $\mathbf{E} = \{(z,w) \in \mathbf{C}^2: |z|^4 + |w|^2 < 1\}$ onto the unit ball in $\mathbf{C}^2$ has the property that it maps points of type four (in the sense of Kohn [**8**]) in the boundary of $\mathbf{E}$ to points of type two in the boundary of the ball. Furthermore, the local branching order of f at these points is two. We prove that this phenomenon holds in general. If M and M' are abstract three-dimensional CR manifolds, and $H: M \to M'$ a CR mapping, there is a notion of multiplicity of H at $p_0 \in M$, for which the type of p_0 is equal to the multiplicity at p_0 times the type of $H(p_0)$. Theorems 1 and 2 state these results more precisely. Theorems 5 and 7 give applications of these results and of the extendability result (Theorem 3) to CR and proper self-mappings.

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