BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 14, Number 1, January 1986

# MIXED HODGE STRUCTURES ON HOMOTOPY GROUPS

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In [**3**] Deligne defined mixed Hodge structures (M.H.S.'s) and showed that the cohomology of every algebraic variety over $\mathbf{C}$ has a natural M.H.S. Morgan [**12**], using Sullivan's minimal models, showed that the rational homotopy Lie algebra and rational homotopy type of every *smooth* variety have natural M.H.S.'s. In this note we announce an extension of mixed Hodge theory to arbitrary varieties and homotopy fibers of morphisms between varieties. The latter is a major step in extending asymptotic Hodge theory to homotopy groups and periods of iterated integrals. The bar construction and Kuo-Tsai Chen's iterated integrals [**1**] provide the link between Hodge theory and homotopy groups. Some of the results announced have been distributed in preprint form [**7**]. Proofs of the results stated will be published elsewhere.

Because the higher homotopy groups of a non-nilpotent topological space are inaccessible to rational homotopy theory, we make the following definition. The *homotopy Lie algebra* of a pointed topological space $(X, x)$ is the graded Lie algebra $\mathfrak{g}_\bullet(X, x)$ where $\mathfrak{g}_0(X, x)$ is the Malcev Lie algebra associated with $\pi_1(X, x)$ and, when $k \geq 1$,

$$\mathfrak{g}_k(X, x) = \begin{cases} \pi_{k+1}(X, x) \otimes \mathbf{Q} & \text{if } (X, x) \text{ is nilpotent,} \\ 0 & \text{otherwise.} \end{cases}$$

The class of nilpotent spaces includes simply connected spaces and topological groups. There is a Hurewicz homomorphism

$$\mathfrak{g}_k(X, x) \to H_{k+1}(V, \mathbf{Q}).$$

THEOREM 1. *If $(V, x)$ is a pointed algebraic variety, then the homotopy Lie algebra of $(V, x)$ has a M.H.S. that is functorial with respect to morphisms of pointed varieties and such that*

(a) *the bracket is a morphism of M.H.S.'s.*

(b) *The Hurewicz homomorphism is a morphism of M.H.S.'s. Moreover, if $(V, W, x)$ is a pair of simply connected varieties, then $\pi_\bullet(V, W, x)$ has a natural M.H.S. and the long exact sequence of the pair is a long exact sequence of M.H.S.'s.*

If $(V, x)$ is simply connected, then the M.H.S. on $\pi_k(V, x)$ does not depend on the basepoint $x$. However, if $V$ is not simply connected, this is not the case.

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Received by the editors April 25, 1985.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 14F35, 14F40; Secondary 55P62.
[1]Supported in part by grants MCS-8201642 and DMS-8401175 from the National Science Foundation.