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Subharmonic functions, vol. 2, by W. K. Hayman. Academic Press, London, 1990, 590 pp., $53.50. ISBN 0-12-334802-1

A function $u : \Omega \to [-\infty, \infty)$, where Ω is a domain in $\mathbb{R}^m$, is said to be subharmonic (s.h.) if it is upper semicontinuous, not identically $-\infty$, and satisfies the sub mean value inequality: its average over the boundary of each ball contained in Ω is greater than or equal to its value at the center. For $m = 1$ the s.h. functions are the convex ones.

S.h. functions were introduced by F. Riesz in the 1920s. They have come to play a central role in several branches of analysis, notably potential and complex function theories. The book under review is a sequel to *Subharmonic functions*, vol. 1, which Hayman co-authored with P. B. Kennedy [HK]. Volume 1 was devoted to development of the rudiments of potential theory, such as solution of the Dirichlet problem for $\Delta u = 0$, and to the basic properties of subharmonic functions, such as the Riesz decomposition theorem. (If u is s.h. in Ω then its distributional Laplacian Δu, known as the Riesz mass, is a locally finite positive measure on Ω, and, loosely speaking, u equals a potential of Δu plus a harmonic function.) The theory expounded there works pretty much the same in $\mathbb{R}^m$ for every $m \geq 2$.

Volume 2, at 590 pages, is twice as long as Volume 1. Its principal aim is to study, in depth, certain families of extremal problems about entire and meromorphic functions of one complex variable. Most of these questions first arose in the early part of the twentieth century. If f is analytic in $\Omega \subset \mathbb{C} = \mathbb{R}^2$ then $\log|f|$ is s.h., while if f is meromorphic then $\log|f|$ is "δ-subharmonic," that is, the difference of two s.h. functions. The problems treated here turn out often to be most naturally posed in the more general s.h. or δ-s.h. context. Thus, the emphasis in Volume 2 is on functions s.h. in all of $\mathbb{C}$, although there are also numerous results for functions in the unit disk of $\mathbb{C}$, as well as some that still hold in $\mathbb{R}^m$ for $m \geq 3$.

One of the book's main themes is the relation between the maximum and minimum values of s.h. functions on circles. Let u be