5. D. Hilbert, *Mathematische Probleme*, Collected Works. III, no. 17, Springer-Verlag, Berlin, 1932, p. 290.
6. F. Hirzebruch, *Hilbert modular surfaces*, Enseign. Math. (2) **71** (1973), 183–281.
7. F. Hirzebruch and D. Zagier, *Intersection numbers on curves on Hilbert modular surfaces and modular forms of Nebentypus*, Invent. Math. **36** (1976), 57–113.
8. M. Rapoport, *Compactifications de l'espace de modules de Hilbert-Blumenthal*, Compositio Math. **36** (1978), 255–335.

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The title may suggest that the book deals with the general theory of transcendental numbers. A complex number $\alpha$ is said to be *algebraic* if it is a root of a polynomial $f(x) = a_nx^n + \cdots + a_1x + a_0$ with rational coefficients and $f(x) \not\equiv 0$. If $\alpha$ is not algebraic, it is called *transcendental.* In 1874, Cantor showed that the set of all algebraic numbers is countable so that transcendental numbers exist. The first rigorous proof of the existence of transcendental numbers was given thirty years earlier by Liouville. We say that $\alpha$ is of *degree* $n$, if the smallest degree of polynomials $f$ as described above equals $n$. Liouville proved the existence of a positive constant $c(\alpha)$ such that every pair of rational integers $p$, $q$ with $q > 0$ and $p/q \neq \alpha$ satisfies

$$\left|\alpha - \frac{p}{q}\right| > \frac{c(\alpha)}{q^n} \quad (n \text{ is degree of } \alpha). \tag{1}$$

It is an easy consequence that numbers with very good rational approximations, such as $\sum_{n=1}^{\infty} 2^{-n!}$, are transcendental. After successive improvements of the exponent $n$ due to Thue (1909), Siegel (1921) and Dyson, Gelfond (1947/1948), Roth (1955)