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CRITICAL BEHAVIOUR OF SELF-AVOIDING WALK IN FIVE OR MORE DIMENSIONS

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ABSTRACT. We use the lace expansion to prove that in five or more dimensions the standard self-avoiding walk on the hypercubic (integer) lattice behaves in many respects like the simple random walk. In particular, it is shown that the leading asymptotic behaviour of the number of n-step self-avoiding walks is purely exponential, that the mean square displacement is asymptotically linear in the number of steps, and that the scaling limit is Gaussian, in the sense of convergence in distribution to Brownian motion. Some related facts are also proved. These results are optimal, according to the widely believed conjecture that the self-avoiding walk behaves unlike the simple random walk in dimensions two, three and four.

1. INTRODUCTION

The self-avoiding walk is a simply defined mathematical model with important applications in polymer chemistry and statistical physics. It serves as a basic example of a non-Markovian stochastic process, but lies beyond the reach of current methods of probability theory. In addition, it poses simply stated combinatorial problems which have not yet met a mathematically satisfactory resolution.

The basic definitions are as follows. An n-step self-avoiding walk ω on the d-dimensional integer lattice $\mathbf{Z}^d$ is an ordered set $\omega = (\omega(0), \omega(1), \dots, \omega(n))$, with each $\omega(i) \in \mathbf{Z}^d$, $|\omega(i+1) - \omega(i)| = 1$ (Euclidean distance), and $\omega(i) \neq \omega(j)$ for $i \neq j$. We always take $\omega(0) = 0$. Thus a self-avoiding walk can be considered as the path of a simple random walk, beginning at the origin, which contains no closed loops. We denote by c_n the number of n-step self-avoiding walks, and for $x \in \mathbf{Z}^d$ we denote by $c_n(x)$ the number of n-step self-avoiding walks for which $\omega(n) = x$. By convention, $c_0 = 1$ and $c_0(x) = \delta_{x,0}$. When x is a nearest

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