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THE TAIT FLYPING CONJECTURE

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ABSTRACT. We announce a proof of the Tait flyping conjecture; the confirmation of this conjecture renders almost trivial the problem of deciding whether two given alternating link diagrams represent equivalent links. The proof of the conjecture also shows that alternating links have no "hidden" symmetries.

In the nineteenth century, the celebrated physicist and knot tabulator P. G. Tait proposed the following conjecture: given reduced, prime alternating diagrams D_1, D_2 of a knot (or link), it is possible to transform D_1 to D_2 by a sequence of *flypes*, where a flype is a transformation most easily described by the pictures of Figure 1 on p. 404.

In performing a flype, the tangle represented by the shaded disc labelled S_A is turned upside-down so that the crossing to its left is removed by untwisting, and a new crossing is created to its right; if the tangle diagram S_A has no crossing, the flype leaves the link diagram unchanged up to isomorphism, whereas if the tangle diagram S_B should have no crossing, the flype amounts merely to a rotation of the complete link diagram about an axis in the projection 2-sphere. During the last few years, some partial results have appeared; in particular it follows from the analysis of [B-S] on arborescent links that any two alternating diagrams of a link which are *algebraic* (i.e. which have Conway basic polyhedron 1*) must be related via a sequence of flypes. A slightly stronger version of this result is set forth in [T4], where the conclusion is obtained for a pair of alternating diagrams only one of which is given as algebraic. It follows from the results of [B-M] that the Tait conjecture holds for link diagrams which are closures of alternating 3-string braid diagrams. K. Murasugi and J. Przytycki [M-P] have proved a number of results on graph polynomials which have lent support to the conjecture. Very recently, A. Schrijver [S] has announced

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