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THE BURAU REPRESENTATION OF THE BRAID GROUP B_n IS UNFAITHFUL FOR LARGE n

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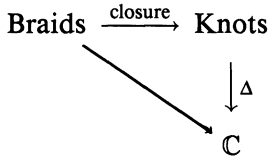
Two fundamental theorems of classical knot theory, by J. Alexander, are that every knot is a closed braid, and second, that a certain procedure (see [1, 2] for both results) assigns to each knot K a polynomial $\Delta(K)$ in T, which only depends on the topological type of the knot once normalized by a power of T to take a positive value at $T = 0$. With this normalization, take T to be a transcendental number.

Since 1926 [2], it has been known that the function Δ: Knots $\to$ $\mathbb{C}$ is not one-to-one. However, it has not become clear whether the composite



which is, up to a constant multiple, a virtual character on the positive braids on n strands and, up to normalization, agrees with the extension to a virtual character on the full braid group B_n, yields a faithful virtual character for all n. This is the natural question if one wishes to know whether Δ is an effective tool for studying braids, or for studying knots viewed as closed braids; and it is equivalent to the question of the faithfulness of the Burau representation. Partial results on the latter question were obtained by Magnus, Magnus-Peluso, Lipshultz, Smythe, and others (see, for instance, Magnus' collected works [3]). The connection between braids and knots is developed in Birman's monograph [4] and the connection with mapping-class groups. Using the connection between braids and knots, V. Jones discovered a more general

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