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EXISTENCE OF NON-SUBNORMAL POLYNOMIALLY HYPONORMAL OPERATORS

RAÚL E. CURTO AND MIHAI PUTINAR

INTRODUCTION

In 1950, P. R. Halmos, motivated in part by the successful development of the theory of normal operators, introduced the notions of subnormality and hyponormality for (bounded) Hilbert space operators. An operator T is subnormal if it is the restriction of a normal operator to an invariant subspace; T is hyponormal if $T^*T \geq TT^*$. It is a simple matrix calculation to verify that subnormality implies hyponormality, but the converse is false. One reason is that subnormality is invariant under polynomial calculus (indeed, analytic functional calculus), while hyponormality is not. If one then defines T to be polynomially hyponormal when $p(T)$ is hyponormal for every polynomial $p \in \mathbf{C}[z]$, the following question arises naturally.

Main Question. Let T be polynomially hyponormal. Must T be subnormal?

Both the class of subnormal operators and the class of hyponormal operators have been the subject of much investigation during the last thirty years or so, and many important developments in operator theory have dealt with them (e.g., the Berger-Shaw Theorem, the singular integral model, and the 2-subscalar model, for hyponormal operators; S. Brown's proof of the existence of nontrivial invariant subspaces, J. Conway and R. Olin's construction of the functional calculus and J. Thomson's description of the spectral picture in the cyclic case, for subnormal operators, etc.;

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