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GLOBAL ORDER FROM LOCAL SOURCES

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1. INTRODUCTION

This article contains introductions to three open problems of significant research interest, taken from number theory, logic, and condensed matter physics. All three problems will be shown to have at their core special cases of one simply-stated optimization problem. Our goal is to use the intuition gained from these three perspectives to direct attention to this common core, which constitutes, in fact, one problem of remarkable depth and importance. We will also show that some of the tools developed in the separate problems are of real value in the others.

Since each of the three problems uses jargon peculiar to its field, we will give an informal introduction to each, together with all relevant definitions, in the following section. However it may be useful to include here a very brief description of each of them to give some idea of our eventual goal.

Our first problem is "sphere packing," in which we consider arrangements of infinitely many unit diameter spheres, each sphere having a variable position in $\mathbf{R}^3$, and try to determine those "optimal" arrangements in which the spheres are disjoint and yet occupy the largest possible fraction of space.

The problem from logic was originally concerned with the "decidability of AEA formulas." It then expanded to the area now known loosely as "tiling theory," in which one analyzes the tilings of the plane which are possible from a given set of tiles. Neighboring tiles must satisfy color (or matching) rules, and we try to optimize agreement of these rules.

From physics we consider the "crystal problem." Here the concern is to understand why real matter seems, experimentally, to

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