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GEODESIC FLOWS, INTERVAL MAPS, AND SYMBOLIC DYNAMICS

ROY ADLER AND LEOPOLD FLATTO

1. Introduction

Geodesic flows and interval maps are two topics in the theory of dynamical systems with a long mathematical history. The first of these seems to have originated with Jacobi who related the flows to the study of Hamiltonian systems (for a detailed description of the connection, see [CFS]). The second arises in diverse settings, such as the modelling of population genetics [Ma] and the frequency count of digits in continued fraction expansions [Bi]. In both subjects the main problem is to describe the distribution of orbits. Thus we wish to know how the geodesics spread over the manifold containing them and how iterates of points under an interval map vary over the interval. Ergodic theory provides answers to these questions, particularly the notions of ergodicity and invariant measure which will be elaborated below.

At first sight the two topics seem unrelated, geodesic flow being a continuous time action and interval map a discrete one. Nevertheless, we shall relate them and their associated symbolic dynamics when the flow takes place on a compact surface of constant negative curvature. In this case we use a graphic approach enabling us to find a series of reductions from geodesic flow to interval map. In relating the topics, we show how each sheds light on the other. We use ergodicity of interval maps to prove ergodicity of flows and conversely. Furthermore, explicit formulas for invariant measures of interval maps can be derived from invariant measures for flows. This fact is interesting as there is a paucity of explicit formulas for invariant measures of interval maps.

The steps in our reduction scheme are known to exist abstractly. However, for the dynamical systems considered here, the reductions are carried out by means of elementary geometry. Our graphic

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