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Large deviations, by Jean-Dominique Deuschel and Daniel W. Stroock. Academic Press, New York, 1989, 300 pp., $34.95. ISBN 0-12-213150-9

The primary concern of the theory of large deviations is the precise estimation of the probabilities of certain classes of rare events. There is usually a natural parameter in the problem which can be assumed to be large. For example, this parameter could denote the size of the system, or if one is dealing with random perturbation of deterministic systems, the noise level could be related to the inverse of this parameter. Either the model or the event or sometimes both depend on this parameter, and the probability usually goes to zero exponentially fast in the parameter. The theory is concerned with the determination of the exact exponential decay rate. Very often the constant can be calculated explicitly in terms of quantities of physical significance. It is the existence of explicit formulae that makes the subject attractive to mathematicians and physicists. Many of the problems of equilibrium and nonequilibrium statistical mechanics have interpretations in terms of the theory of large deviations.

Here are two typical examples of the theory. Let $x_1, x_2, \dots x_n$ be n independent identically distributed (i.i.d.) random variables having $F(x)$ for their common distribution function. Cramer [1] proved in 1937 that

$$\lim_{n\to\infty} \frac{1}{n} \log P\left[a \le \frac{x_1 + \cdots + x_n}{n} \le b\right] = -c$$

exists for $-\infty \le a < b \le \infty$ and the constant c can be explicitly calculated in terms of F according to the following recipe:

$$c = \inf_{a\le x\le b} I(x)$$

where

$$I(x) = \sup_\theta[\theta x - \log M(\theta)]$$

and

$$M(\theta) = \int e^{\theta x} dF(x).$$

The Scandinavian school in the 1930s was interested in the calculation of risks in the insurance business and Cramer's theorem was