BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 2, April 1991

0273-0979/91 $1.00 + $.25 per page

Lectures on hyponormal operators, by M. Martin and M. Putinar. Birkhauser, Boston, 1989, 305 pp., $80.00. ISBN 3-7643-2329-9

A *hyponormal operator* is a bounded operator T on a Hilbert space $\mathscr{H}$ such that $T^*T \geq TT^*$. This rather innocent definition was introduced by Paul Halmos [11] in 1950 and generalizes the concept of a normal operator (where $T^*T = TT^*$). Why not consider the condition $T^*T \leq TT^*$, you ask? In fact, people do; such operators are called *cohyponormal*. (Needless to say, the two theories are related, though one is not a trivial adaptation of the other since many properties do not travel well when taking adjoints.) The important thing is that there is a prominent example of a hyponormal operator, the unilateral shift. If l^2 is the Hilbert space of square summable sequences and T is defined on l^2 by $T(a_0, a_1, \ldots) = (0, a_0, a_1, \ldots)$, then T is called the *unilateral shift* and is the most basic of hyponormal operators.

Normal operators are completely understood. Indeed, it is possible to define a complete set of unitary invariants for normal operators; equivalently, it is possible to give a model for an arbitrary normal operator. Specifically, in the case that the underlying Hilbert space $\mathscr{H}$ is separable, given any compactly supported regular Borel measure μ on the complex plane and a Borel function m defined on $\mathbf{C}$ with values in $\{\infty, 0, 1, 2, \ldots\}$ such that $m = 0$ off the support of μ, there is a canonically associated normal operator $N_{\mu,m}$ and each normal operator is unitarily equivalent to one of these models. That is, for each normal operator N there is a μ, such a function m, and a unitary operator U with $N = U^*N_{\mu,m}U$. Moreover, two such models $N_{\mu,m}$ and $N_{\nu,n}$, are unitarily equivalent if and only if $[\mu] = [\nu]$ (that is, μ and ν are equivalent measures in the sense that they have the same sets of measure 0) and $m = n$ a.e. $[\mu]$. The details of this can be found in [10], §IX.10. In the case of a nonseparable Hilbert space the