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SPECTRAL THEORY OF REINHARDT MEASURES

RAÚL E. CURTO AND KEREN YAN

Let μ be a finite positive Borel measure on $\mathbf{C}^n$ $(n \geq 1)$, with compact support K, let $P^2(\mu)$ be the norm closure in $L^2(\mu)$ of the algebra of complex polynomials in $z_1, \ldots, z_n$, and let $M_z = (M_{z_1}, \ldots, M_{z_n})$ be the n-tuple of multiplication operators by the coordinate functions $z_1, \ldots, z_n$ acting on $P^2(\mu)$. M_z is the universal model for cyclic subnormal n-tuples of operators acting on a separable Hilbert space. For $n = 1$, the spectral and algebraic properties of M_z have been the focus of extensive study (see [Con] for a survey account of the basic results in this area). One important instance, the case $d\mu(\mathrm{re}^{i\theta}) = d\rho(r) \times \frac{d\theta}{2\pi}$ (where ρ is a positive Borel measure on $[0, +\infty)$), gives rise to the class of subnormal weighted shifts, via Berger's Theorem [Con, III.8.16]. Here, the spectral picture of M_z admits a very simple description:

(i) $\sigma(M_z)$, the spectrum of M_z, equals $D_\mu := \{\lambda \in \mathbf{C}: |\lambda| \leq \sup\{|z|: z \in K\}\}$;

(ii) The Fredholm domain of M_z is $\mathbf{C}\backslash\partial D_\mu$; and

(iii) $\operatorname{index}(M_z - \lambda) = -1$ whenever $\lambda \in \operatorname{int}(D_\mu)$.

The circular symmetry of weighted shifts, reflected in the above description, appears in several variables in the notion of *Reinhardt* set; $F \subseteq \mathbf{C}^n$ is Reinhardt if $F = \tau^{-1}(\tau(F))$, where $\tau: \mathbf{C}^n \to \mathbf{R}_+^n$ is given by $z \to (|z_1|, \ldots, |z_n|)$. Correspondingly, a compactly supported positive Borel measure μ is Reinhardt if it admits a decomposition $d\mu(\mathrm{re}^{i\theta}) = d\rho(r) \times d\theta/(2\pi)^n$, where ρ is a positive Borel measure on $\mathbf{R}_+^n$. For instance, volumetric Lebesgue measure on a complete bounded Reinhardt domain $\Omega \subseteq \mathbf{C}^n$ is a Reinhardt measure, in which case $P^2(\mu)$ is actually $A^2(\Omega)$, the Bergman space over Ω.

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