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HYPERGEOMETRIC FUNCTIONS ON COMPLEX MATRIX SPACE

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0. Introduction

In [6] we presented the general foundations for a theory of hypergeometric functions of matrix argument over real division algebras. In this note, we further develop the fine structure of these functions over the complex field, including series expansions, integral representations, asymptotics, differential equations, addition formulas, multiplication formulas, summation theorems, transformation properties, etc. Especially important in this paper are the *operator-valued hypergeometric functions*, required for (nonspherical) expansions such as addition formulas by the noncommutativity of matrix multiplication. These functions generalize the operator-valued Bessel functions studied in [5].

Hypergeometric functions of matrix argument arise naturally in applications ranging from multivariate statistics, quantum physics, and molecular chemistry, to harmonic analysis, group representations, and number theory. (See the references in [6].) These diverse applications amplify the need to develop the fine structure systematically and to the greatest extent possible.

We briefly review the definition of hypergeometric functions of matrix argument from [6]. Let $\mathbf{F}$ be the real field, the complex field, or the quaternions. Denote by S the space of all $n \times n$ Hermitian matrices $s = s^*$ over $\mathbf{F}$, on which the group $G = GL(n, \mathbf{F})$ of invertible $n \times n$ matrices g over $\mathbf{F}$ acts by $s \mapsto g^* s g$. Then $K = \{k \in G : k^* k = 1\}$ is the isotropy subgroup of the identity matrix 1, the open cone P in S of positive-definite $n \times n$ matrices is the orbit under G of 1, and P can be identified with the symmetric space $K \backslash G$. A functions f on S is *K-invariant* if

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