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MACDONALD'S CONSTANT TERM CONJECTURES FOR EXCEPTIONAL ROOT SYSTEMS

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ABSTRACT. We announce proofs of Macdonald's constant term conjectures for the affine root systems $S(F_4)$ and $S(F_4)^\nu$. We also give an algorithm for deciding the conjectures for the remaining root systems $S(E_6)$, $S(E_7)$, and $S(E_8)$ and prove that the constant term in question can indeed be expressed in closed form. Combined with previous work of Zeilberger-Bressoud, Kadell, and Gustafson, our results imply that Macdonald's conjectures are true *in form* for any root system, and the complete truth of Macdonald's conjectures is a finite number of mips away.

1. INTRODUCTION AND RESULTS

Root systems and reflection groups occupy a central position in Lie theory [Hu1], finite groups [C] and other branches of mathematics, and are also an intriguing object of study for their own sake [Hu2, B]. In 1972, Macdonald [Ma1] proved a series of "formal" identities, one for every affine root system, that for the simplest affine root system $S(A_1)$ specialized to Jacobi's triple product formula. These formulas had numerous number-theoretic applications [Ma1, D] and also constituted the "tip of the iceberg" that led Victor Kac to the theory of representations of Kac-Moody algebras [Kac, pp. xiii, xiv]. In 1982, Macdonald [Ma2] conjectured a collection of constant term identities that constituted "finite forms" generalizations of his celebrated identities. The most general of these conjectures, which was obtained by jointly generalizing an earlier conjecture of his (qM) and a conjecture of Morris [Mo] (see also [A]) for G_2, has the form

(qM-M)

$$\text{C.T.} \prod_{\alpha\in R^+} (x^\alpha ; q^{u_\alpha})_{k_\alpha} (q^{u_\alpha} x^{-\alpha} ; q^{u_\alpha})_{k_\alpha} = \text{ a certain explicit product.}$$

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