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# THE CONJECTURE OF LANGLANDS AND RAPOPORT FOR SIEGEL MODULAR VARIETIES

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Frequently one attaches to an arithmetic object a zeta function whose analytic behavior reflects deep properties of the object. For example, attached to $\mathbf{Q}$ there is the Riemann zeta function, whose behavior (especially the position of its zeros) contains information about the distribution of the prime numbers. Dedekind attached a zeta function to every algebraic number field, and Hasse and Weil attached a zeta function to every algebraic variety over a number field. A central point of Langlands's philosophy is that these zeta functions should be among the $L$-functions that he attaches to automorphic representations of reductive groups.

A natural first testing ground for this idea is the family of Shimura varieties. For the simplest Shimura varieties, the elliptic modular curves, the results of Eichler and Shimura can be interpreted as demonstrating its validity. In general, to define a Shimura variety, one needs a reductive group $G$ over $\mathbf{Q}$ and a finite disjoint union $X$ of bounded symmetric domains on which $G(\mathbf{R})$ acts. For example, to define the elliptic modular curves, one takes $G$ to be $GL_2$ and $X$ to be the complex plane minus the real axis. The Shimura variety is then a family of varieties $\mathrm{Sh}(G, X)$ over $\mathbf{C}$, but a deep result says that it has a canonical model over a number field $E$. For almost all primes $v$ of $E$, this model reduces to a family of nonsingular varieties $\mathrm{S}_v$ over the finite residue field $k(v)$ at $v$, and the zeta function of the Shimura variety is defined in terms of the number of points of $S_v$ with coordinates in the finite fields containing $k(v)$.

Let $\mathbf{F}$ be the algebraic closure of $k(v)$. The starting point of Langlands's program for realizing the zeta function of $\mathrm{Sh}(G, X)$ as an automorphic $L$-function is to obtain an explicit description of the set of points on the Shimura variety with coordinates in $\mathbf{F}$, together with the action of the Frobenius automorphism of $\mathbf{F}$ and

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