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THE SELBERG ZETA FUNCTION AND SCATTERING POLES FOR KLEINIAN GROUPS

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In this note we present a polynomial bound on the distribution of poles of the scattering operator for the Laplacian on certain hyperbolic manifolds M^n of infinite volume. The motivation is to understand more fully the geometry of the poles of the scattering operator. The proof uses the relationship between poles of the scattering operator and zeros of the Selberg zeta function for geodesic flow on M^n.

Recall that the classical Selberg zeta function $Z(s)$ [30] is a meromorphic function which describes the lengths $\ell(\gamma)$ of closed geodesics γ on a compact surface S:

$$Z(s) = \prod_{\gamma} \prod_{m=1}^{\infty} \{1 - \exp(-(s+m)\ell(\gamma))\} \tag{1}$$

where the product over γ runs over primitive closed geodesics. Crude estimates on the distribution of lengths show that $Z(s)$ is analytic for $\Re(s) > 1$; an application of Selberg's trace formula shows that $Z(s)$ extends to a meromorphic function on $\mathbf{C}$ with trivial zeros at the integers $1, 0, -1, \ldots$ together with spectral zeros at the numbers s_k where $s_k(1-s_k)$ is an eigenvalue of the Laplacian on S [11, 20, 30].

Here we will be concerned with the Selberg zeta functions, introduced by Patterson [25], for certain noncompact hyperbolic manifolds M^n of infinite hyperbolic volume. These zeta functions are defined by an infinite product similar to (1) (identical when $n = 2$); they share important features with the classical zeta function. In particular, their analytic structure is closely tied to the spectrum of the Laplacian on M^n. However, the Laplacian

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