RESEARCH ANNOUNCEMENTS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 2, April 1991

DISTRIBUTION RIGIDITY FOR UNIPOTENT ACTIONS ON HOMOGENEOUS SPACES

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In this paper we study distributions of orbits of unipotent actions on homogeneous spaces.

More specifically, let $\mathbf{G}$ be a real Lie group (all groups in this paper are assumed to be second countable) with the Lie algebra $\mathfrak{G}$, Γ a discrete subgroup of $\mathbf{G}$ and $\pi: \mathbf{G} \to \Gamma\backslash\mathbf{G}$ the projection $\pi(\mathbf{g}) = \Gamma\mathbf{g}$, $\mathbf{g} \in \mathbf{G}$. The group $\mathbf{G}$ acts by right translations on $\Gamma\backslash\mathbf{G}$, $(x, \mathbf{g}) \to x\mathbf{g}$, $x \in \Gamma\backslash\mathbf{G}$, $\mathbf{g} \in \mathbf{G}$. We say that Γ is a lattice in $\mathbf{G}$ if there is a finite $\mathbf{G}$-invariant measure on $\Gamma\backslash\mathbf{G}$.

Let $\mathbf{U}$ be a subgroup of $\mathbf{G}$ and $x \in \Gamma\backslash\mathbf{G}$. We say that the closure $\overline{x\mathbf{U}}$ of the orbit $x\mathbf{U}$ in $\Gamma\backslash\mathbf{G}$ is *homogeneous* if there is a closed subgroup $\mathbf{H} \subset \mathbf{G}$ such that $\mathbf{U} \subset \mathbf{H}$, $\mathbf{xHx}^{-1} \cap \Gamma$ is a lattice in $\mathbf{xHx}^{-1}$, $\mathbf{x} \in \pi^{-1}\{x\}$, and $\overline{x\mathbf{U}} = x\mathbf{H}$. If these conditions are satisfied, we shall say that $\overline{x\mathbf{U}}$ is homogeneous with respect to $\mathbf{H}$.

Definition 1. A subgroup $\mathbf{U} \subset \mathbf{G}$ is called *topologically* rigid if given any lattice $\Gamma \subset \mathbf{G}$ and any $x \in \Gamma\backslash\mathbf{G}$ the closure of the orbit $x\mathbf{U}$ in $\Gamma\backslash\mathbf{G}$ is homogeneous.

A subgroup $\mathbf{U} \subset \mathbf{G}$ is called unipotent if for each $\mathbf{u} \in \mathbf{U}$ the map $\mathrm{Ad}_{\mathbf{u}}: \mathfrak{G} \to \mathfrak{G}$ is a unipotent automorphism of $\mathfrak{G}$.

Raghunathan's Topological Conjecture. *Every unipotent subgroup of a connected Lie group* $\mathbf{G}$ *is topologically rigid.*

Received by the editors July 26, 1990.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 22E40.
Partially supported by the NSF Grant DMS-8701840.