# FIFTY YEARS OF EIGENVALUE PERTURBATION THEORY

## BARRY SIMON

ABSTRACT. We highlight progress in the study of eigenvalue
perturbation theory, especially problems connected to quantum
mechanics. Six models are discussed in detail: isoelectronic
atoms, autoionizing states, the anharmonic oscillator, double
wells, and the Zeeman and Stark effects. Berry's phase is also
discussed

## 1. INTRODUCTION

Eigenvalue perturbation theory has its roots in work of Lord
Rayleigh in acoustics at the turn of the century, and of Schrödinger
in his fundamental series founding quantum theory in the twenties.
In recognition of their contributions, the series is called Rayleigh-
Schrödinger series. But the mathematical foundations were only
set by Rellich just over fifty year ago [40]. In the developments
since, a critical role was played by Tosio Kato, both in his papers
and in his classic book [32].

The Kato-Rellich theory concerns general abstract operator
theory—analytic operators in the regular case and asymptotic se-
ries in some nonregular cases. It turns out that many of the ex-
amples of interest in quantum physics do not fit into the scheme
of regular perturbation theory. While some do meet the criteria
of Kato's asymptotic perturbation theory, mere asymptoticity is
not a very satisfying state of affairs and one would hope for ad-
ditional insight from perturbation theory in suitable cases. So the
last twenty-five years have seen detailed analysis by mathematical
physicists of specific operator combinations often without abstract
operator theoretic analogs.