RESEARCH ANNOUNCEMENTS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 24, Number 1, January 1991

THREE RIGIDITY CRITERIA FOR PSL(2, R)

CHRISTOPHER BISHOP AND TIM STEGER

Statement of results

Let G be $\mathrm{PSL}(2, \mathbf{R})$, the quotient of the group of 2×2 real matrices with determinant one by its two element center, $\{\pm I\}$. By a *lattice subgroup* of G we mean a discrete subgroup such that the space of cosets G/Γ has finite volume. A familiar example of a lattice subgroup is $\mathrm{PSL}(2, \mathbf{Z})$, the subgroup of matrices in $\mathrm{PSL}(2, \mathbf{R})$ with integer entries. Let Γ be an abstract group and let ι_1 and ι_2 be two inclusions of Γ in G, each having a lattice subgroup as its image. We say ι_1 and ι_2 are *equivalent* if there is some (continuous) automorphism α of G so that $\iota_2 = \alpha \circ \iota_1$. This paper describes three closely related criteria for the equivalence of ι_1 and ι_2: one analytic, one representation theoretic, and one geometric.

If G were $\mathrm{PSL}(n, \mathbf{R})$ for some $n > 2$, or indeed if it were any connected simple Lie group with trivial center except for $\mathrm{PSL}(2, \mathbf{R})$, then the Mostow rigidity theorem (see [M1, M2, Ma, P]) would assert that ι_1 and ι_2, as described above, are necessarily equivalent: a given abstract group Γ could be embedded in G as a lattice subgroup in at most one way (up to automorphisms of G). This remarkable theorem is false for $\mathrm{PSL}(2, \mathbf{R})$. Indeed, the

Received by the editors January 16, 1990 and, in revised form, August 1, 1990.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 22E40, 22E45.
Key words and phrases. Fuchsian groups, lattices, Poincaré series, representations, rigidity, Teichmüller space.
Both authors are partially supported by the NSF.

0273-0979/91 \$1.00 + \$.25 per page