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The asymptotic solution of linear differential systems, applications of the Levinson theorem, by M. S. P. Eastham. Clarendon Press, Oxford, 1989, vii + 240 pp., $63.00. ISBN 0-19-853299-7

The earliest results on the asymptotic approximation to solutions of ordinary differential equations can be traced back to the latter part of the ninteenth century and the early part of the twentieth century in the works of Poincaré, Kneser, Horn, Bôcher, Dunkel, Birkhoff, and others. These early works had as their focus the asymptotic approximation of solutions by means of divergent series which are formal solutions of the differential equations. However, by taking any fixed finite number of terms of the formal divergent series, one gets an explicit function which is asymptotic to a solution of the differential equation as the independent variable approaches a point on the extended real line. In these two senses the word "asymptotic" has two slightly different meanings. During the last forty to fifty years a good deal of work on asymptotic solutions has concentrated on the second meaning. The results so obtained may not be as precise as the consideration of asymptotic series, but they have the enormous advantage that much wider classes of differential equations and systems can be dealt with. These wider classes of equations and asymptotic solutions are of importance for applications, which has been the main driving force behind these investigations.

In the book under review, the author deals only with the second aspect of asymptotic solution. In particular, the bulk of the book concentrates on two important applications of a remarkable asymptotic theorem of N. Levinson which the latter established in 1948 in an essentially definitive form. The Levinson theorem deals with $n \times n$ systems of ordinary differential equations of the form

$$Y'(x) = [A + V(x) + R(x)]Y(x), \qquad 0 < x < \infty,$$

where A is a matrix with n distinct characteristic roots, $V(x) \to 0$ as $x \to \infty$, and

$$\int_0^\infty |V'(x)|\,dx < \infty, \qquad \int_0^\infty |R(x)|\,dx < \infty.$$