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DISCRETE ANALOGUES OF SINGULAR RADON TRANSFORMS

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The purpose of this paper is to describe recent results we have obtained in finding discrete analogues of the theory of singular integrals on curves, or more generally of "singular Radon transforms," at least in the translation-invariant case. Our theorems are related to estimates for certain exponential sums that arise in number theory; they are also connected with Bourgain's recent maximal ergodic theorem [2, 3]. The detailed proofs of our results are quite lengthy, and will appear elsewhere. Here we shall limit ourselves to stating the main conclusions, and sketching the motivation and background. We take this opportunity to acknowledge our indebtedness to Guido Weiss and A. De la Torre, whose suggestions were the starting point of this research.

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The theory of singular Radon transforms may be thought of as a natural extension of the Calderón-Zygmund theory of singular integrals. It had as its origin some problems related to parabolic differential equations, and was developed further because of its real-variable consequences, in particular differentiation theory along lower-dimensional varieties; see [10]. Later its relevance to several complex variables and its connection with analysis on nilpotent Lie groups were brought out; (see [6, 7, 8]). Here we begin by stating one of the main known results in the $\mathbf{R}^n$ setting for a basic model problem.

Let K be a Calderón-Zygmund convolution kernel on $\mathbf{R}^k$ (here k need not equal n). Then K is defined away from the origin, satisfies the estimates $|K(x)| \leq A|x|^{-k}$, $|\nabla k(x)| \leq A|x|^{-k-1}$, and the cancellation property: $\int_{R\leq|x|\leq\gamma R} K(x)dx = 0$, for some $\gamma > 1$, and all $0 < R < \infty$.

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