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$L^p \to L^{p'}$ ESTIMATES FOR TIME-DEPENDENT SCHRÖDINGER OPERATORS

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MAIN RESULTS

Let $H_0 = -\Delta$, where $\Delta = (\partial/\partial x_1)^2 + \cdots + (\partial/\partial x_n)^2$ is the Laplacian in $\mathbf{R}^n$. For $t \in \mathbf{R}$, one can define $u(\cdot, t) = e^{itH_0} f$ using the spectral theorem. The function one obtains is the solution to the time-dependent Schrödinger equation

$$\begin{cases} i\partial u/\partial t + H_0 u = 0 \\ u(x, 0) = f(x). \end{cases} \tag{1}$$

Since the kernel of e^{itH_0} is $(4\pi it)^{-n/2} e^{|x-y|^2/4it}$, it is clear that the solution is dispersive in the sense that

$$\|u(\cdot, t)\|_{L^{p'}(\mathbf{R}^n)} \le Ct^{-n(1/p-1/2)} \|f\|_{L^p(\mathbf{R}^n)}, \quad t > 0, \tag{2}$$

if

$$1 \le p \le 2, \quad \text{and} \quad 1/p + 1/p' = 1. \tag{3}$$

It is well known that the local decay estimates (2) are useful in studying nonlinear Schrödinger equations (see [8, §XI.13], [11]). On the other hand little seems to be known when one replaces the free operator H_0 by more general Hamiltonians

$$H = -\Delta + V(x), \tag{4}$$

even when the potential V is in $C_0^\infty(\mathbf{R}^n)$. Obviously, one has to assume that H has no bound states for an estimate like (2) to hold for $u = e^{itH} f$. If in addition $n \ge 3$ and if one assumes that there are no half-bound states (i.e., zero resonances) the best-known decay estimates seem to be

(5)

$$\|\langle x\rangle^{-\alpha} u(\cdot, t)\|_{L^2(\mathbf{R}^n)} \le Ct^{-n/2} \|\langle x\rangle^{\alpha'} f\|_{L^2(\mathbf{R}^n)}, \quad \langle x\rangle = \sqrt{1 + |x|^2},$$

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