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SUBCRITICALITY, POSITIVITY, AND GAUGEABILITY OF THE SCHRÖDINGER OPERATOR

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1. INTRODUCTION

We investigate properties of the Schrödinger operator $H := -(\Delta/2) + V \geq 0$ in $R^d (d \geq 3)$ in the following three aspects:

(I) Subcriticality: Intuitively, the idea is that if $H \geq 0$ is subcritical, then it should be possible to perturb H by small perturbations and still keep its nonnegativity. More precisely, we have the following assertions:

(a) For any $q \in B_c$ (B_c denotes the class of bounded Borel functions with compact support), there exists an $\varepsilon > 0$ such that $-(\Delta/2) + V + \varepsilon q \geq 0$.
(b) There exists a function $q \in B_c$, $q \leq 0$ and $q \not\equiv 0$ a.e. such that $-(\Delta/2) + V + q \geq 0$.

There have been two other definitions of subcriticality:

(c) (B. Simon [7]) There exists $\beta > 0$ such that $-(\Delta/2) + (1+\beta)V \geq 0$.
(d) (M. Murata [6]) There exists a positive Green function $G^H(\cdot, \cdot)$ for H.

(II) Strong Positivity:

(e) There exists a positive solution $u > 0$ of $Hu = 0$ with the limit: $\lim_{|x|\to\infty} u(x) > 0$.
(f) There exists a solution u of $Hu = 0$ with $c' \geq u \geq c > 0$.
(g) There exists a solution u of $Hu = 0$ with $u \geq c > 0$.

(III) Gaugeability: Let $\{X_t : t \geq 0\}$ be the Brownian motion in R^d and let E^x denote the expectation over the Brownian paths starting from $x \in R^d$. Put $u_0(x) := E^x[\exp(-\int_0^\infty V(Xs)\,ds)]$.

(h) $u_0(x) \not\equiv \infty$ in R^d.
(i) $u_0(x)$ is bounded in R^d.

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