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A NEW UPPER BOUND FOR THE MINIMUM OF AN INTEGRAL LATTICE OF DETERMINANT 1

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ABSTRACT. Let Λ be an n-dimensional integral lattice of determinant 1. We show that, for all sufficiently large n, the minimal nonzero squared length in Λ does not exceed $[(n+6)/10]$. This bound is a consequence of some new conditions on the theta series of these lattices; these conditions also enable us to find the greatest possible minimal squared length in all dimensions $n \leq 33$. In particular, we settle the "no-roots" problem: There is a determinant 1 lattice containing no vectors of squared length 1 or 2 precisely when $n \geq 23$, $n \neq 25$. There are also analogues of all these results for codes.

1. INTRODUCTION

The problem of classifying n-dimensional integral lattices of determinant 1 has been studied by Magnus, Mordell, Ko, Witt, Kneser, Niemeier and others [4, Chapters 1, 16, and 17]. The lattices Λ of this type for which the minimal norm

$$\min\{u \cdot u \colon u \in \Lambda, \quad u \neq 0\}$$

takes its highest possible value μ are of the greatest interest. It was shown in [7] that for even lattices (those in which $u \cdot u$ is always even), the minimal norm is at most $2[n/24]+2$, while for odd lattices (those in which $u \cdot u$ is sometimes odd) the corresponding bound is $[n/8]+1$ [7, 11]. These are the bounds one would expect from the dimension of the space of available theta series. In fact, it is known that μ differs from these bounds by an amount that tends to infinity with n, so that equality can hold for only finitely many lattices [7]. In the odd case, the bound holds with equality for precisely 12 lattices, the highest dimension of which is 23 [2, 4, Chapter 19]. As to lower bounds, it is known that both even and odd lattices exist in which the minimal norm is asymptotically at least $n/2\pi e$ [4, Chapter 7; 10].

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