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THE JUMP IS DEFINABLE IN THE STRUCTURE OF THE DEGREES OF UNSOLVABILITY

S. BARRY COOPER

Recursion theory deals with computability on the natural numbers. A function f from $\mathbf{N}$ to $\mathbf{N}$ is computable (or recursive) if it can be calculated by some program on a Turing machine, or equivalently on any other general purpose computer. A major topic of interest, introduced in Post [23], is the notion of relative difficulty of computation. A function f is computable *relative* to a function g if after equipping the machine with a black box subroutine that provides the values of g, there is a program (which now may call g via the subroutine) which computes f. In this case we write $f \leq_T g$. Two functions are Turing equivalent if each is computable relative to the other; the equivalence classes are called Turing degrees. These degrees form a partial ordering $\mathscr{D}$ under the induced reducibility relation $\leq$. The structural analysis of the partial ordering $\mathscr{D}$ has been a major area of research in recursion theory since the pioneering paper of Kleene and Post [14].

Kleene and Post proved a number of results on the structure of $\mathscr{D}$ including the embeddability of arbitrary countable partial orders into $\mathscr{D}$, and obtained partial results on extendability of a given embedding to a larger domain. This line of investigation was pursued by many people over the next twenty-five years, culminating in essentially complete solutions of these problems, and a characterization of the possible ideals of the structure $\mathscr{D}$ (see Lachlan and Lebeuf [16] and Lerman [17], [18]).

Kleene and Post also considered the enriched structure $\mathscr{D}'$ equipped with the "jump operator", denoted $'$, which is a canonical operation on degrees which takes each degree $\mathbf{d}$ to a strictly

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