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GORENSTEIN LOCAL HOMOMORPHISMS

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INTRODUCTION

A Noetherian local ring is the algebraic version of a ring of germs of functions defined in neighborhoods of some point of an algebraic (or analytic) variety. Accordingly, local rings are naturally classified by the complexity of the singularity they describe, with the simplest class consisting of the *regular* rings, which correspond to nonsingular points. On the singular side a natural boundary is provided by the *Cohen-Macaulay* rings: beyond them pathological (that is, geometrically unpredictable) behavior becomes a common phenomenon.

During the last three decades much of the work in commutative algebra has concentrated on rings whose singularities interpolate between these two extremes. One of the most important developments early in that period was the discovery of the intermediate class of *Gorenstein* rings by Bass and Grothendieck. These authors demonstrated that Gorenstein rings provide a perfect framework for the investigation of duality phenomena, and this is the main reason behind their ubiquitous appearance in commutative algebra and algebraic geometry. They also noted that among the Gorenstein singularities one finds all *local complete intersections*, which describe points of transversal intersection of hypersurfaces.

The purpose of this note is to introduce some of the results of [2], where a *relative* theory of Gorenstein singularities is systematically developed. There are several aspects to our approach. First, it gives a unified treatment of hitherto unrelated relative Gorenstein notions, such as that of *flat* homomorphisms whose fibres are Gorenstein rings (Grothendieck), and *surjective* homomorphisms whose kernels are generated by regular sequences, or more generally, are Gorenstein ideals (Buchsbaum and Eisenbud). Next, it contains the theory of Gorenstein rings as the *absolute* case, that

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