BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 22, Number 2, April 1990

0273-0979/90 $1.00 + $.25 per page

Lattices with Unique Complements, by V. N. Salii. Translations of Mathematical Monographs, Vol. 69. Translated by G. A. Kandall and B. Silver. American Mathematical Society, Providence, RI, 1988, ix+113 pp., $55.00. ISBN 0-8218-4522-5

1. Introduction

It is a good idea to introduce a branch of mathematics by discussing one of its interesting topics. If the discussion is thorough, then it can also provide the specialist with an up-to-date research survey.

Introducing lattice theory via the theory of uniquely complemented (UC, for short) lattices is especially appropriate. UC lattices form a natural foundation for Boolean algebras, a field well known to all mathematicians. In fact, the fundamental problem of this field originates in Boolean algebras: Is every UC lattice Boolean?

This branch of lattice theory is well-motivated, surprisingly deep, and its techniques have found widespread application.

2. The story

While in natural sciences we usually start the history of a field with "The ancient Greeks already knew that ...," in lattice theory "ancient history" refers roughly to the years 1900–1935. Pioneers of lattice theory introduced the lattice concept as a tool in the axiomatization of Boolean algebras: C. S. Peirce, E. Schröder, and especially, E. V. Huntington. Independently, R. Dedekind introduced lattices and modular lattices as abstractions of ideals of algebraic numbers (rings).

Some of these early results are interesting and nontrivial. For instance, in 1904, E. V. Huntington characterized Boolean algebras as complemented lattices in which the complementation is pseudocomplementation, that is, $a \wedge x = 0$ implies that $x \leq a'$.

This and some similar results led Huntington to the conjecture that a UC lattice is Boolean (that is, distributive).

In "The Dilworth Theorems. Selected Papers of Robert P. Dilworth" (edited by K. P. Bogart, R. Freese, and J. P. S. Kung, Birkhäuser Verlag, to appear), the article by M. E. Adams on UC