BOOK REVIEWS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 22, Number 2, April 1990

0273-0979/90 $1.00 + $.25 per page

Mathematical intuitionism. Introduction to proof theory, by A. G. Dragalin. Translations of Mathematical Monographs, Vol. 67. Translated by E. Mendelson. American Mathematical Society, Providence, R. I., 1988, ix+228 pp., $75.00. ISBN 0-8218-4520-9

A common misconception among mathematicians is to think of intuitionistic mathematics as "mathematics without the law of the excluded middle" (the law asserting that every statement is either true or false). From this point of view, intuitionistic mathematics is a proper subset of ordinary mathematics, and doing your mathematics intuitionistically is like doing it with your hands tied behind your back.

Another more realistic viewpoint is to regard intuitionistic logic, and the mathematics based on that logic, as the logic of sets with some structure, rather than of bare sets. Traditional examples are sets growing in time (as in Kripke semantics [9]), or set with some recursive structure (as in Kleene's realizability interpretation [7]), or sets continuously varying over some fixed parameter space. Universes of such sets are perfectly suitable for developing mathematics, but one is often *forced* to use intuitionistic logic.

This is a small price to pay for the many new phenomena that can be observed in such universes: For example, in some such universes one has a "recursive axiom of choice," which states that for any sequence $\{A_n\}_n$ of nonempty subsets $A_n \subset \mathbf{N}$ there is a *recursive* choice function $f\colon \mathbf{N} \to \mathbf{N}$ selecting an element $f(n)$ from