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DIFFERENTIABILITY OF ENTROPY FOR ANOSOV AND GEODESIC FLOWS

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INTRODUCTION

Topological and metric entropies are among the most important global invariants of smooth dynamical systems. Topological entropy characterizes the total exponential complexity of the orbit structure with a single number. Metric entropy with respect to an invariant measure gives the exponential growth rate of the statistically significant orbits. The knowledge of entropies, especially in low-dimensional cases, provides a wealth of quantitative structural information about the system. Such information includes the growth rate of the number of periodic orbits [K 1], "large horseshoes" [K 3], the growth rate of the volume of cells of various dimensions ([Mn] for geodesic flows, [Y] in general), ergodic components, and factors with very stochastic behavior [Pe, **Si**], etc.

Unfortunately, because the entropies are defined in global asymptotic terms, *a priori*, we do not expect them to change smoothly when the system is perturbed, even in a very nice topology like C^∞ or C^ω (real analytic).

There are several results concerning the regularity of entropy for general diffeomorphisms and flows on compact manifolds. Misiurewicz [Mi 1] constructed examples to show that the topological entropy, $h_{\text{top}}\colon \text{Diff}_\infty(M^n) \to \mathbf{R}$ is not continuous for $n \geq 4$. It seems unknown, although unlikely, whether entropy is continuous for $n = 3$. Yomdin [Y] and Newhouse [N] have proved that $h_{\text{top}}\colon \text{Diff}_\infty(M^n) \to \mathbf{R}$ is upper-semicontinuous for $n \geq 2$. Katok [K 1, K 3] has shown that for surfaces, $h_{\text{top}}\colon \text{Diff}_2(M^2) \to \mathbf{R}$ is lower-semicontinuous. By combining these two results, one sees

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