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A SYMPLECTIC GEOMETRY APPROACH TO GENERALIZED CASSON'S INVARIANTS OF 3-MANIFOLDS

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1. In 1985 lectures at MSRI, Andrew Casson introduced an integer valued invariant $\lambda(M)$ for any oriented integral homology 3-sphere M^3. This invariant has many remarkable properties; detailed discussions of some of these can be found in an exposé by S. Akbulut and J. McCarthy (see [AM]). Roughly, $\lambda(M)$ measures the 'oriented' number[3] of irreducible representations of the fundamental group $\pi_1(M)$ in $SU(2)$.

In the preceding article of this journal, Kevin Walker [W] described results of his thesis which yield an invariant $\lambda(M^3)$ of an arbitrary oriented rational homology 3-sphere (RHS: $H_1(M^3, Q) = 0$) which extends Casson's invariant. His creative methods give generalizations of the properties which Casson had earlier shown for oriented integral homology 3-spheres (IHS: $H_1(M^3, \mathbf{Z}) = 0$). For homology lens spaces, Boyer and Lions [BL] have independently obtained an inductive definition of $\lambda(M^3)$. Earlier, a different extension of Casson's invariant to certain rational homology spheres, which does not equal Walker's invariant, had been studied by S. Boyer and A. Nicas [BN]. In all of the above works, one is considering only representations into $SU(2)$.

The present announcement solves the problem, which has been emphasized by Atiyah [A], of producing generalizations of Casson's invariant to invariants of M^3 that would roughly measure the 'oriented' number of representations of $\pi_1(M)$ in $G = SU(n)$, for each $n \geq 2$. We introduce $\lambda_G(M^3)$, an invariant which is defined for an arbitrary oriented rational homology 3-sphere (RHS).

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[3] Following [W] we do not divide by 2 as in [A-M].