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AN EXTENSION OF CASSON'S INVARIANT TO RATIONAL HOMOLOGY SPHERES

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In 1985, Andrew Casson defined an invariant $\lambda(M)$ of an oriented integral homology 3-sphere M [C, AM]. This invariant can be thought of as counting the number of conjugacy classes of nontrivial representations $\pi_1(M) \to SU(2)$, in the sense that the Lefschetz number of a map counts the number of fixed points. Casson proved the following three properties of λ.

(i) If $\pi_1(M) = 1$, then $\lambda(M) = 0$.

(ii) Let N be the complement of a knot in a homology sphere and let $N_{1/n}$ denote N Dehn surgered along one meridian and n longitudes (see below for terminology). Then

$$\lambda(N_{1/n}) = \lambda(N) + n\Delta''_N(1),$$

where $\Delta''_N(t)$ is the second derivative of the Alexander polynomial of N.

(iii) $4\lambda(M)$ is congruent modulo 16 to the μ-invariant (see below) of M.

This paper describes an extension of Casson's methods to the case where M is a rational homology 3-sphere, including generalizations of (ii) and (iii). (This extension is different from the one given in [BN].) In addition, an alternate definition of λ, using the generalized Dehn surgery formula, is given (Theorem 1).

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