BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 22, Number 2, April 1990

MAPPING PROBLEMS IN COMPLEX ANALYSIS AND THE $\bar{\partial}$-PROBLEM

S. BELL

1. INTRODUCTION

Mark Twain's most famous short story, *The Celebrated Jumping Frog of Calaveras County*, was translated into many languages during his lifetime, including French. The French did not think the story was funny. Twain, in order to discover whether it was a flaw in the French persona or a flaw in the translation that rendered his hilarious story a flop, had the French translation translated word for word back into English. The French persona was exonerated; indeed the retranslation was not funny. In this paper, I will translate some results from one complex variable into the language of several variables, and then back again to one variable. The end result will differ from the original. I hope that the new perspective will enhance, rather than detract from, our understanding of the original.

Translating a result from one complex variable to several is more involved than merely saying, "Now let $n > 1$." Indeed, many arguments in one variable use the special relationship that exists between harmonic and holomorphic functions in the plane. In several variables, harmonic functions do not enjoy an elevated status; they are almost never mentioned. Thus, in several variables, a substitute must be found for the Laplace operator and the functions it annihilates. Generally, it is the $\bar{\partial}$-operator which replaces the Laplacian. In one variable, the $\bar{\partial}$-operator is given by $\partial/\partial\bar{z} = \frac{1}{2}(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y})$. An excellent place to see classical one variable results proved using $\bar{\partial}$-techniques is in the first chapter of Hörmander's book on several complex variables [23]. For exam-

Received by the editors August 22, 1988.

1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 30C35, 32H99, 35N15.

Research supported by NSF grant DMS 8420754. This paper is based on an Invited Address given by the author at the AMS meeting in Indianapolis, April, 1986.