it is clear that the development of all areas of real algebraic geometry will benefit greatly from the existence of *Géométrie algébrique réelle.*

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Abelian l-adic representations and elliptic curves, by Jean-Pierre Serre. Addison-Wesley Publ. Co., Reading, Mass., 1989, 140 pp., ISBN 0-201-09384-7

Addison-Wesley has just reissued Serre's 1968 treatise on l-adic representations in their Advanced Book Classics series. This circumstance presents a welcome excuse for writing about the subject, and for placing Serre's book in a historical perspective.

The theory of l-adic representations is an outgrowth of the study of abelian varieties in positive characteristic, which was initiated by Hasse and Deuring (see, e.g., [3, 1]) and continued in Weil's 1948 treatise [12]. Over the complex field $\mathbf{C}$, an abelian variety A of dimension g may be viewed as an (algebrizable) complex torus W/L, where $L \approx \mathbf{Z}^{2g}$ is a lattice in the $\mathbf{C}$-vector space W of dimension g. The classical study of A relies heavily on the lattice L, which is intrinsically the first homology group $H_1(A, \mathbf{Z})$. However, the quotients L/nL (for $n \geq 1$) have a purely algebraic definition. Indeed, over $\mathbf{C}$ the quotient L/nL is canonically the group

$$A[n] = \{P \in A \mid n \cdot P = 0\}$$

of n-division points on A. Over an arbitrary field K, one defines $A[n]$ as the group of points on A (with values in a separable closure $\overline{K}$ of K) of order dividing n. When n is prime to the characteristic of K, $A[n]$ is a free $\mathbf{Z}/n\mathbf{Z}$-module of rank $2g = 2\dim A$, just as in the classical case. Moreover, the module $A[n]$ carries natural commuting actions of the Galois group $\mathrm{Gal}(\overline{K}/K)$ and the ring $\mathrm{End}_K(A)$ of K-endomorphisms of A. Most information provided by L can be extracted from the collection of groups $A[l^\nu]$ $(\nu \geq 1)$, where l is a fixed prime which is different from the characteristic of K.