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Triangulated categories in the representation theory of finite dimensional algebras, by Dieter Happel. Cambridge University Press, Cambridge, New York, New Rochelle, Melbourne, Sydney, 1988. (London Mathematical Society Lecture Notes Series, vol. 119), ix + 208 pp., $24.95. ISBN 0-521-33922-7

The concept of a triangulated category seems to have appeared in the sixties, and to go back to suggestions of Grothendieck. The basic reference is the article of Verdier [26]. The most famous example is the derived category $D^b(\mathscr{A})$ of bounded complexes over an abelian category $\mathscr{A}$. It is constructed as follows. Let $C^b(\mathscr{A})$ denote the (abelian) category of bounded complexes over $\mathscr{A}$. In many situations, occurring for instance in algebraic geometry, complexes are determined only up to quasi-isomorphisms, that is, morphisms of complexes which induce an isomorphism in cohomology. We would then like to replace $C^b(\mathscr{A})$ by a new category in which such complexes could be declared isomorphic. The first step in the construction comes from the observation that in such situations, morphisms of complexes are only determined up to homotopy. It is thus reasonable to replace $C^b(\mathscr{A})$ by the homotopy category $K^b(\mathscr{A})$, whose objects are the same as those of $C^b(\mathscr{A})$, but whose morphisms are the homotopy classes of morphisms of complexes. The homotopy category $K^b(\mathscr{A})$ is not abelian in general. It enjoys a weaker structure, that of a triangulated category. In this structure, short exact sequences are replaced by what are called distinguished triangles. Finally, one obtains the derived category $D^b(\mathscr{A})$ from $K^b(\mathscr{A})$ by declaring all the quasi-isomorphisms invertible, or, more formally, by localising $K^b(\mathscr{A})$ relative to the multiplicative system of quasi-isomorphisms (using the calculus of fractions developed in [11]). The category $D^b(\mathscr{A})$ inherits from $K^b(\mathscr{A})$ the structure of a triangulated category. Since its introduction, this concept was used in many problems of algebraic geometry and homological algebra, most notably in duality theory (see, for instance, [18, 6 or 20]).

Recently, it was shown that the derived categories of certain categories of coherent sheaves are equivalent to the derived category of bounded complexes of finitely generated modules over certain