In sum, the book may be highly recommended (with the caveat above) to beginners who wish a bird's-eye view of this broad and beautiful, but sometimes deep and sophisticated theory.

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Unit groups of classical rings, by Gregory Karpilovsky, Clarendon Press, Oxford, 370 pp., $98.00. ISBN 0-19-853557-0

Call a ring unitary if it has an identity element under multiplication. If R is a unitary ring, then there are several groups and monoids that are naturally associated with R. Among these are the additive group $(R, +)$ of R (that is, the group on the set R with operation the operation of addition defined on the ring R), the multiplicative monoid $(R, \cdot)$ of R, and the multiplicative group $U(R)$ of units of R. (A unit of R is an element that has a multiplicative inverse in R; for example 1 and -1 are the units of the ring of integers.) Ring theorists have long been interested in the interplay and relations that exist between the algebraic structures R, $(R, +)$, $(R, \cdot)$ and $U(R)$. Clearly R nominally determines the other three structures. What about the converse? To what extent do one or more of the structures $(R, +)$, $(R, \cdot)$ and $U(R)$ determine R? A different kind of question concerns realization: for example, given an abelian group G and a group H, can G and H be realized as the additive and unit groups, respectively, of a unitary ring R, and if so, how many realizations are there, to within isomorphism? To illustrate this last question, suppose $G = Z$, the infinite cyclic group. If G is the additive group of a unitary ring R, and if g is a generator for G, then the multiplication on R is completely determined by the integer k, where $g^2 = kg$; moreover, $k = \pm 1$ since R is unitary. Since $(-g)^2 = (-k)(-g)$, where $-g$ is also a generator for G, it follows that R is isomorphic to the ring of integers, so H must be cyclic of order two in order for the pair (G, H) to be realizable. In a similar vein, Chapter 6 of the book under review determines the unitary rings R for which $U(R)$ is cyclic. Natural variants on these themes arise if one restricts to rings or groups that satisfy a given condition E. For example, early work by Fuchs, Szele