BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 22, Number 1, January 1990

THE ERROR TERM IN NEVANLINNA THEORY. II

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Nevanlinna theory [Ne] was created to give a quantitative measure of the value distribution for meromorphic functions, for instance to measure the extent to which they approximate a finite number of points. We view a meromorphic function as a holomorphic map $f : \mathbf{C} \to \mathbf{P}^1$ into the projective line. The theory has various higher dimensional analogues, of which we shall later consider maps $f : \mathbf{C}^n \to X$ where X is a projective complex manifold of dimension n.

We first deal with the classical case of Nevanlinna with $n = 1$. Let $a \in \mathbf{P}^1$. By a **Weil function** associated with a we mean a continuous function

$$\lambda_a : \mathbf{P}^1 - \{a\} \to \mathbf{R}$$

having the property that in some open neighborhood of a there exists a continuous function α such that if z is a local coordinate at a, then

$$\lambda_a(z) = -\log|z-a| + \alpha(z).$$

The difference between two Weil functions is a continuous (and therefore bounded) function on $\mathbf{P}^1$. A Weil function roughly measures the distance from a. As usual, for real $x > 0$ define $\log^+(x) = \max(\log x, 0)$. Let z be the standard coordinate on $\mathbf{C}$. Nevanlinna takes the functions

$$\lambda_a(z) = \log^+ 1/|z-a| \quad \text{if } a \neq \infty,$$
$$\lambda_a(z) = \log^+ |z| \quad \text{if } a = \infty.$$

One defines the corresponding **mean proximity function**

$$m_f(\lambda_a, r) = \int_0^{2\pi} \lambda_a(f(re^{i\theta}))\frac{d\theta}{2\pi}.$$

One usually writes $m_f(a, r)$ instead of $m_f(\lambda_a, r)$ since a definite

Received by the editors April 25, 1989 and, in revised form, July 11, 1989.
1980 *Mathematics Subject Classification* (1985 *Revision*). Primary 11J68, 30D35, 32H30.