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ON THE REGULARITY THEORY OF FULLY NONLINEAR PARABOLIC EQUATIONS

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INTRODUCTION

Recently M. Crandall and P. L. Lions [3] developed a very successful method for proving the existence of solutions of nonlinear second-order partial differential equations. Their method, called the theory of viscosity solutions, also applies to fully nonlinear equations (in which even the second order derivatives can enter in nonlinear fashion). Solutions produced by the viscosity method are guaranteed to be continuous, but not necessarily smooth. Here we announce smoothness results for viscosity solutions. Our methods extend those of [1]. We obtain Krylov-Safonov (i.e. C^α estimates [8]), $C^{1,\alpha}$, Schauder ($C^{2,\alpha}$) and $W^{2,p}$ estimates for viscosity solutions of uniformly parabolic equations in general form. The results can be viewed as *a priori* estimates on the classical C^2 solutions. Our method produces, in particular, regularity results for a broad new array of nonlinear heat equations, including the Bellman equation [6]:

$$u_t - \sup_{\alpha\in A}[a_{ij}^\alpha(x, t)u_{ij} + b_i^\alpha(x, t)u_i + c^\alpha(x, t)u - g^\alpha(x, t)] = 0.$$

On the other hand, in the special case of linear equations, to which our method of course also applies, our proofs are much easier than the classical estimates for classical solutions, and also produce new results in this long-and well-studied field. For elliptic equations, similar results were obtained by Caffarelli [1], in the case that the equations do not involve the term Du.

We consider the following equation for a real-valued function u:

$$(1) \qquad u_t - F(D^2u, Du, u, x, t) = 0,$$

where $u_t = \partial u/\partial t$, $D^2u = (\partial^2 u/\partial x_i x_j)$, $Du = (\partial u/\partial x_i)$. Classically, there are two ways of attacking the problem of regularity

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