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SYMMETRY BREAKING IN EQUIVARIANT BIFURCATION PROBLEMS

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1. INTRODUCTION

In both equivariant bifurcation theory [GSS, especially Chapter XIII] and physical theories of spontaneous symmetry breaking (for example, the Higgs-Landau theory [M]), there is the problem of determining the symmetries, stabilities and branching patterns for solutions of equations equivariant under a compact Lie group G. Very few general results and techniques are known for the analysis of this problem, though versions of a Maximum Isotropy Subgroup Conjecture have been conjectured, to the effect that generically all solution branches have maximal isotropy (see for example [G, M]). General results of this type are of particular interest for applications on account of the inherent complexity of the structure of isotropy subgroups, invariants and equivariants for G-representations. In this note, we announce several new results for the general study of the symmetries and branching patterns for a large class of G-equivariant bifurcation problems. In particular, we give new counterexamples to the Maximal Isotropy Subgroup Conjecture and present examples where one can get precise information on the branching patterns. Our methods also show that one can get quite detailed information on these problems without full knowledge of the G-equivariants. To simplify our exposition, we assume G finite.

Let V be a finite dimensional real Hilbert space and $G \to \mathbf{O}(V)$ be an absolutely irreducible representation of the finite group G. Let G act on $V \times \mathbb{R}$ by $g \cdot (x, \lambda) = (g \cdot x, \lambda)$ and let $\mathscr{X} = C^\infty_G(V \times \mathbb{R}, V)$ be the space of smooth G-equivariant maps of $V \times \mathbb{R}$ to V. Give $\mathscr{X}$ the C^∞-topology; subsets of $\mathscr{X}$ are given the induced topology. Each $f \in \mathscr{X}$ defines a one-parameter family $(f_\lambda)_{\lambda \in \mathbb{R}}$ of equivariant vector fields on V. We have $f(0, \lambda) = 0$, $\lambda \in \mathbb{R}$. These are the *trivial zeros* of f. We study zeros of f bifurcating off the trivial zeros. Now $D_1 f(0, \lambda) = \sigma_f(\lambda) Id_V$,

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