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SMOOTH EXTENSIONS FOR A FINITE CW COMPLEX

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The C^*-algebra extensions of a topological space can be made into an abelian group which is naturally equivalent to the K-homology group of odd dimension [1] which has a close relation with index theory and is one of the starting points of KK theory [8].

The C_p-smoothness of an extension of a manifold was introduced in [3, 4], where C_p denotes the Schatten-von Neumann p-class [5]. We generalize the notion of C_p-smoothness to a finite CW complex and obtain necessary and sufficient conditions for an extension of a finite CW complex to be C_p-smooth modulo torsion.

The notion of C_p-smooth extensions is one of the motivations for Connes' cyclic cohomology. In [2] Connes constructs a Chern map from $KK(C(M), \mathbf{C})$ to the cyclic cohomology of $C^\infty(M)$, and proves that this Chern map is a surjection modulo torsion. One consequence of the even counterpart of our main results is that this Chern map is a graded surjection modulo torsion. We will make this statement precise in Theorem 3.

Let H be an infinite dimensional complex separable Hilbert space. By $L(H)$ and $K(H)$ we shall denote the C^*-algebra of bounded operators and compact operators on H, respectively, and $Q(H)$ will denote the quotient $L(H)/K(H)$ with canonical surjection $\pi : L(H) \to Q(H)$. For X a compact metrizable space an extension $\tau \in \operatorname{Ext}(X)$ of the algebra $C(X)$ by $K(H)$ is defined by a unital $*$ monomorphism $\tau : C(X) \to Q(H)$ [1].

Definition 1. Let M be a smooth compact manifold (perhaps with boundary) and let $C^\infty(M)$ denote the $*$-algebra of all smooth functions on M. A $\tau \in \operatorname{Ext}(M)$ is C_p-smooth if there exists a $*$-linear map $\rho : C^\infty(M) \to L(H)$ such that $\rho(ab) - \rho(a)\rho(b) \in C_p$ and $\pi \circ \rho = \tau | C^\infty(M)$.

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