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RIGIDITY AND OTHER TOPOLOGICAL ASPECTS OF COMPACT NONPOSITIVELY CURVED MANIFOLDS

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ABSTRACT. Let M be a compact connected Riemannian manifold whose sectional curvature values are all nonpositive. Let Γ denote the fundamental group of M. We prove that any homotopy equivalence $f: N \to M$ from a compact closed manifold N is homotopic to a homeomorphism, provided that $m \geq 5$ where $m = \dim M$. We show that the surgery L-group $L_{k+m}(\Gamma, w_1)$ is isomorphic to the set of homotopy classes of maps $[M \times I^k \operatorname{rel} \partial, G/\mathrm{TOP}]$, where I^k is the k-dimensional cube (with $k > 0$). We also show that the Whitehead group $\mathrm{Wh}(\Gamma)$, the projective class group $\tilde{K}_0(Z\Gamma)$, and the lower K-groups $K_{-n}(Z\Gamma)$, $n \geq 1$, are all isomorphic to the one element group. The higher K-groups $K_n(Z\Gamma)$, $n \geq 0$, are computed up to rational isomorphism type. All of these results have previously been obtained by the authors in the case that the sectional curvature values of M are strictly negative (cf. [7, 8, 9, 10]).

In all the following results we let M denote a compact connected Riemannian manifold all of whose sectional curvature values are nonpositive, and we let Γ denote the fundamental group of M.

Theorem 1. *If $h: N \to M$ is a homotopy equivalence from a compact closed manifold N, and if $\dim(M) \geq 5$, then there is a homotopy of h to a homeomorphism.*

Let $\mathscr{P}(M)$ denote the semisimplicial space of stable topological pseudo-isotopies of M. For any stratified fibration $p: E \to B$ we let $\mathscr{P}(E; p)$ denote the semisimplicial space of compactly supported stable topological pseudo-isotopies on E which have arbitrarily small control in B (defined in [23]). If $f: E \to M$ is a continuous map then denote by $F: \mathscr{P}(E; p) \to \mathscr{P}(M)$ the map which is induced by f.

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