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Iwasawa theory of elliptic curves with complex multiplication, by Ehud de Shalit. Perspectives in Mathematics, vol. 3, Academic Press, Orlando, 1987, ix + 154 pp., $19.50. ISBN 0-12-210255-x

One of the most fascinating aspects of number theory and arithmetic algebraic geometry is the deep and mysterious connection between arithmetic and analysis. One example of this is the formula for the residue of the zeta function of a number field F,

$$\lim_{s\to 1}(s-1)\zeta_F(s) = \frac{2^{r_1}(2\pi)^{r_2}hR}{w\sqrt{d}} \tag{1}$$

where r_1 (resp. r_2) is the number of real (resp. complex) embeddings of F, h is the class number of F, R is the regulator (a determinant of logarithms of global units) of F, w is the number of roots of unity in F, and d is the discriminant of F.

Another, more modern example deals with elliptic curves. If E is an elliptic curve defined over a number field F (i.e., E is a curve defined by an equation $y^2 = x^3 - ax - b$ with $a, b \in F$ and $4a^3 - 27b^2 \neq 0$), then E has an L-function and various arithmetic invariants. The fundamental object of arithmetic interest is the set $E(F)$ of points on E with coordinates in F; $E(F)$ has a natural abelian group structure and by the Mordell-Weil theorem this group is finitely generated. The L-function is defined by an Euler product over primes $\mathfrak{p}$ of F,

$$L(E,s) = \prod_{\mathfrak{p}} L_{\mathfrak{p}}(\mathbf{N}\mathfrak{p}^{-s})$$

where $L_{\mathfrak{p}}(T)$ is a polynomial in T of degree at most 2, whose coefficients depend on the reduction of E modulo $\mathfrak{p}$. The conjecture of Birch and Swinnerton-Dyer states that

$$\operatorname{rank}_{\mathbf{Z}} E(F) = \operatorname{ord}_{s=1} L(E,s) \tag{2}$$

and further, if we denote this common value by r, the conjecture expresses $\lim_{s\to 1}(s-1)^{-r}L(E,s)$ in terms of other invariants of E, with a formula analogous to (1).