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Ergodic theory and differentiable dynamics, by Ricardo Mañé, Ergebnisse der Mathematik und ihrer Grenzgebiete, 3. Folge, Bd. 8, Springer-Verlag, Berlin, 1987, xi + 317 pp., $82.00. ISBN 0-387-15278-4

In 1931 G. D. Birkhoff published the proof of one of the most profound theorems of this century [**B**]. This theorem, which has come to be known as the Birkhoff ergodic theorem, is remarkable in several ways. It is the only recent instance which comes to mind of a single theorem giving rise to a whole new branch of mathematics. Moreover it is one of those rare theorems whose content and significance can largely be understood by nonmathematicians.

The motivation for the ergodic theorem came from the work of Boltzmann and Gibbs on statistical mechanics. The mathematical question arising from their work was under what conditions the limit

$$\lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n} f(T^i(x))$$

exists and is independent of $x \in X$, where $f : X \to R$ is a real valued function on a space X and $T : X \to X$ is a transformation. This limit is the average value of the function f along the forward orbit of the transformation T.

Birkhoff's theorem concerns the case when (X, μ) is a finite measure space, f is measurable, and T is a measurable transformation for which the equation $A = T^{-1}(A)$ is never satisfied unless A has measure 0 or full measure. Such transformations are called *ergodic*. The theorem is often paraphrased by saying that for ergodic transformations the time average equals the space average. In other words if we consider the transformation T as a dynamic which occurs every unit of time, then for almost all starting points $x \in X$ the average value of the function f on the orbit of x as it evolves through time exists and is equal to $\int_X f d\mu$, the average value of the function f on the space X. Intuitively, if we consider the case when f is 1 on a measurable set A and 0 elsewhere, then this says that a typical particle x in an ergodic system will spend a proportion of its time in A equal to the proportion of the total volume in A. Except for the technical concept of measure 0 implicit in the conclusion about almost all x, this is easily explainable to a nonmathematician.