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TOEPLITZ C^*-ALGEBRAS OVER PSEUDOCONVEX REINHARDT DOMAINS

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Multivariable Toeplitz operators, acting on Hardy or Bergman spaces over domains in $\mathbf{C}^n$, occur in connection with elliptic boundary value problems [**1**], weighted shift operators [**6**] and problems in function theory of several complex variables [**2**]. If the underlying domain is strictly pseudoconvex [**4**], of finite type [**1, 11**] or symmetric [**13**], the associated Toeplitz operators (with continuous symbol) are essentially commutative or at least generate a solvable C^*-algebra of finite length. In particular, the Toeplitz C^*-algebra is of type I.

In this note we describe the Toeplitz C^*-algebra of *pseudoconvex Reinhardt domains* Ω, using a finite composition series which is geometrically characterized by "boundary foliations" associated with the complex geometry of Ω. Whenever these foliations are of "irrational type," we obtain Toeplitz C^*-algebras which are not of type I (this can happen for domains with smooth boundary). We also announce an index theory for these non-type I Toeplitz C^*-algebras and give some applications to the theory of proper holomorphic mappings. For concreteness, we explain here the case $n = 2$.

Let Ω be a bounded pseudoconvex complete Reinhardt domain (in $\mathbf{C}^2$), with closure $\overline{\Omega}$. By [**8**], these domains are the natural domains of convergence of power series and are characterized by the condition that $(u, v) \in \Omega$ whenever $|u| \leq |z|$, $|v| \leq |w|$ for some $(z, w) \in \Omega$ or $|u| = |z_1|^\lambda |w_1|^{1-\lambda}$, $|v| = |z_2|^\lambda |w_2|^{1-\lambda}$ for some $(z_1, w_1) \in \Omega$, $(z_2, w_2) \in \Omega$ and $0 < \lambda < 1$. We may assume that Ω is *normalized*, i.e., Ω is contained in the bidisk $\mathbf{D}^2$ and contains the coordinate axes $V := \{(z, w) \in \mathbf{D}^2 \colon zw = 0\}$. Then the "logarithmic domain" $C := \{(x, y) \in \mathbf{R}^2 \colon (e^x, e^y) \in \Omega\}$ is an unbounded convex open set contained in the third quadrant and ∂C is a concave curve. Let $\overline{C}$ denote the closure of C in $\mathbf{R}^2$ and let $\partial^j(C)$ be the union of all j-dimensional *faces* of $\overline{C}$ (e.g., $\partial^2(C) = \overline{C}$ and $\partial^0(C)$ consists of all extreme points).

Given a face F of $\overline{C}$, denote by L_F the linear subspace of the same dimension parallel to F. For any point $t = (\xi, \eta)$ in the 2-torus $\mathbf{T}^2$, consider the leaf $t_F := \{(\xi e^{2\pi i x}, \eta e^{2\pi i y}) \colon (x, y) \in L_F\}$ generated by F through t. This gives a foliation $\mathscr{F}_F$ of $\mathbf{T}^2$, with corresponding foliation C^*-algebra (cf. [**5**]) denoted by $C^*(\mathscr{F}_F)$. For $F = \overline{C}$, $\mathscr{F}_F$ has just one leaf ($\mathbf{T}^2$ itself) and $C^*(\mathscr{F}_F)$ is *-isomorphic to the ideal $\mathscr{K}$ of compact operators. For

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